1 Melting and Solidification

$Q = m[(T_{\text{fus}} - T^{\ominus})C_{P,(\text{s})} + \Delta_{\text{fus}}H + (T_{\text{sh}} - T_{\text{fus}})C_{P,(\text{l})}]$

$T_{\text{sh}} \equiv T_{\text{fus}} +^{\circ}$ superheat

$Y_{\text{c}} = \frac{m_{\text{shipped}}}{m_{\text{poured}}} = \frac{m_{\text{casting (s)}}}{m_{\text{riser (l)}} + m_{\text{casting (l)}} + m_{\text{gating}}}$

1.1 Solidification Volume Change

$\{T = T_{\text{sh}}\}: \quad V_{(\text{sh})} = \frac{m}{\rho_{(\text{l})} + \Delta_{\text{sh}}\rho_{(\text{l})} \cdot (T_{\text{sh}} - T)}$

$\{T = T_{\text{fus}}\}: \quad V_{(\text{l})} = \frac{m}{\rho_{(\text{l})}}$

$\{T = T^{\ominus}\}: \quad V^{\ominus}_{(\text{s})} = \frac{m}{\rho^{\ominus}_{(\text{s})}}$

$\underline{\Delta}_{\text{T}}V = \frac{V_{(\text{sh})} - V^{\ominus}_{(\text{s})}}{V_{(\text{sh})}} \qquad \underline{\Delta}_{\text{l}\to\text{s}}V = \frac{V_{(\text{s})} - V_{(\text{l})}}{V_{(\text{s})}} \qquad V_{(\text{s})} = V_{(\text{l})}\left(1 + \underline{\Delta}_{\text{l}\to\text{s}}V\right)$

2 Chvorinov's Rule

$M_{\text{c}} \equiv \left(\frac{V}{A_S}\right)_{\text{metal}} \qquad \alpha_{\text{mould}} \equiv \frac{k}{\rho \hat{C}_P} \qquad$ RoT: $M_{\text{riser}} \geq 1.1 M_{\text{casting}}$

$-\ell_2 = \ell_1 + \ell_1\alpha(T_{\text{fus}} - T^{\ominus})$

2.1 Sand

$t_{\text{s}} = B_{\text{s}} M_{\text{c}}^2$

$B_{\text{s}} \equiv \frac{\pi\alpha_{\text{mould}}}{4k^2_{\text{mould}}}\rho^2_{\text{metal (l)}}\frac{[\Delta_{\text{fus}}H + C_{P,\text{metal (l)}}(T_{\text{pour}} - T_{\text{fus}})]^2}{(T_{\text{fus}} - T^{\ominus})^2}$

2.2 Permanent

$t_{\text{s}} = B_{\text{s}} M_{\text{c}}$

$B_{\text{s}} \equiv \rho_{\text{metal (l)}}\frac{\Delta_{\text{fus}}H + C_{P,\text{metal (l)}}(T_{\text{pour}} - T_{\text{fus}})}{h_{\text{metal:mould}}(T_{\text{fus}} - T_{0,\text{mould}})}$

3 Cylindrical Risers

$M_{\text{r}} \equiv \frac{V}{A_S} \qquad A_S = 2\pi R^2 + \frac{2V}{R}$

RoT: $\hbar = 2\varnothing$, but $\hbar = \varnothing \Rightarrow \eta = 1$

$M_{\text{r, top}} = \left.\frac{\pi R^2\hbar}{\pi R^2 + 2\pi R}\right|_{\hbar = 4R = 2\varnothing} \geq \frac{2\varnothing}{9}$

$M_{\text{r, side}} = \left.\frac{\pi R^2\hbar}{2\pi R^2 + 2\pi R}\right|_{\hbar = 4R = 2\varnothing} \geq \frac{\varnothing}{5}$

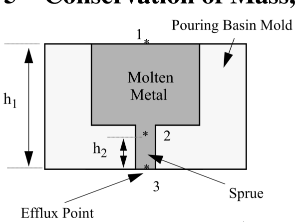
$\ell_{\text{n}} \leq \frac{\varnothing_{\text{r}}}{2} \qquad \varnothing_{\text{n}} \leq 1.2\ell_{\text{n}} + 0.1\varnothing_{\text{r}}$

4 Steel Zones of Effect

$\text{EE} = 2\delta \qquad \text{RE} = 1.5\delta \qquad \text{ECE} = \delta \qquad \text{MCE} = 2\text{EE} = 4\delta$

$N_{\text{r}} = \frac{\ell - 2\text{EE}}{2\text{RE} + \varnothing_{\text{r}}}$ without chill

5 Conservation of Mass, Energy, & Momentum

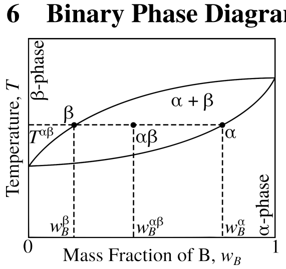


$\tilde{H} = z + \frac{\vec{v}^2}{2g} + \frac{P}{\rho g} + \frac{\hat{F}}{g} \qquad \dot{V}_1\rho = \dot{V}_2\rho \qquad A_{\sigma 1}\vec{v}_1 = A_{\sigma 2}\vec{v}_2$

$\vec{v}_3 = \sqrt{2g\hbar_1} \qquad P_2 = P_3 - \hbar_2\rho \qquad \vec{v}_2 = \sqrt{2g(\hbar_1 - \hbar_2)}$

$A_{\sigma 2}\vec{v}_2 = A_{\sigma 3}\vec{v}_3 \qquad t_{\text{fill}} = \frac{V_{\text{cavity}}}{A_{\sigma 3}\vec{v}_3}$

6 Binary Phase Diagram Lever Rule



$w^{\alpha}(T^{\alpha\beta}) = \frac{w_B^{\alpha\beta} - w_B^{\beta}}{w_B^{\alpha} - w_B^{\beta}} \qquad w^{\beta}(T^{\alpha\beta}) = \frac{w_B^{\alpha} - w_B^{\alpha\beta}}{w_B^{\alpha} - w_B^{\beta}}$

7 Prerequisite Material

7.1 Statics

$A = \iint dA \qquad m = \iint_A \sigma\, dA$

$S_x = \iint_A y\, dA \quad S_y = \iint_A x\, dA \quad M_x = \iint_A y\sigma\, dA \quad M_y = \iint_A x\sigma\, dA$

$C_x = \frac{S_y}{A} \qquad C_y = \frac{S_x}{A} \qquad \bar{x} = \frac{M_y}{m} \qquad \bar{y} = \frac{M_x}{m}$

$I_{Sx} = \iint_A y^2\, dA \qquad I_{Sy} = \iint_A x^2\, dA \qquad I_{Sz} = I_{Sx} + I_{Sy}$

$I_{Mx} = \iint_A y^2\sigma\, dA \qquad I_{My} = \iint_A x^2\sigma\, dA \qquad I_{Mz} = I_{Mx} + I_{My}$

$I_{Mx} = I_{M\bar{x}} + m\delta^2 \qquad I_{Mx} = mk_x^2 \qquad k_x = \sqrt{\frac{I_{Mx}}{A}}$

$I_{Pxy} = \iint_A xy\, dA \qquad I_{Pxz} = \iint_A xz\, dA \qquad I_{Pyz} = \iint_A yz\, dA$

7.2 Dynamics

$\vec{\tau} = I\vec{\alpha} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad |\vec{\tau}| = |\vec{r}||\vec{F}|\sin\theta$

$W = \vec{\tau}\theta \qquad E_{\text{k}} = \frac{1}{2}I\omega^2 \qquad \text{P} = \vec{\tau}\omega$

$\vec{L} = \vec{r} \times \vec{P} \qquad \vec{L} = I\vec{\omega} \qquad \vec{\omega} \equiv \frac{\vec{r} \times \vec{v}}{r^2} \qquad \vec{M} = \vec{r} \times \vec{F}$

7.3 Mechanics of Materials

$\sigma_{\text{e}} = \frac{\vec{F}}{A_{\sigma 0}} \qquad \sigma = \frac{\vec{F}}{A_{\sigma}} = \sigma_{\text{e}}(1 + \varepsilon_{\text{e}})$

$\varepsilon_{\text{e}} = \frac{\Delta\ell}{\ell_0} \qquad \varepsilon = \ln\frac{\ell}{\ell_0} = \ln(1 + \varepsilon_{\text{e}})$

$E = \frac{\sigma}{\varepsilon_{\text{e}}} = \frac{\vec{F}\ell_0}{A_{\sigma}\Delta\ell}$

$\nu = -\frac{\varepsilon_{\text{transverse}}}{\varepsilon_{\text{axial}}}$

$G = \frac{\tau_{xy}}{\gamma_{xy}} = \frac{\vec{F}\ell_0}{A_{\sigma}\Delta x} = \frac{E}{2(1+\nu)}$

$K = -V_0\frac{\mathrm{d}P}{\mathrm{d}V} = \rho_0\frac{\mathrm{d}P}{\mathrm{d}\rho}$

$K_{\text{I}} = Y\sigma\sqrt{\pi\ell_{\text{crack}}}$

7.4 Selected Nomenclature

C	Centroid	m
E	Young's modulus	Pa
G	Shear modulus	Pa
I_{Sx}	Moment of area (2nd) / area mt. of inertia	m^4
I_{Mx}	Moment of mass (2nd) / mass mt. of inertia	$\text{kg}\,\text{m}^2$
I_{Pxy}	Product moment of inertia	m^4
K	Bulk modulus	Pa
K_{I}	Fracture toughness, plane-strain	$\text{Pa}\,\text{m}^{1/2}$
k	Radius of gyration	m
M_x	Moment of mass (1st)	kg m
$\vec{M}$	Moment, bending	N m
S_x	Moment of area (1st)	m^3
$\bar{x}$	Centre of mass	m
Y	Geometric factor	-
γ_{xy}	Shear strain	-
ε_{e}	Strain, engineering	-
ε	Strain, true	-
ν	Poisson's ratio	-
σ_{e}	Stress, engineering	Pa
σ	Stress, true	Pa
τ_{xy}	Shear stress	Pa

References

K. Rundman, *Principles of Metal Casting*. Houghton, MI, USA: Michigan Technological University.

`https://engineeringstatics.org/`